# Strategic Majoritarian Voting with Propositional Goals

Agents express **propositional goals** over **binary issues** to reach a collective decision.
We study the **strategy-proofness** of three generalizations of the **majority rule**.
We also study the **computational complexity** of finding a successful manipulation.

## Framework

- A set $\mathcal{N}$ of $n$ agents has to decide over a set $\mathcal{I}$ of $m$ binary issues (**no integrity constraint**)
- Every agent $i$ has a propositional formula $\gamma_i$ as her goal, whose models are in the set $\text{Mod}(\gamma_i)$
- $m_i(j) = (m^0_{ij}, m^1_{ij})$ indicates the number of 0s and 1s for issue $j$ in the models of $\gamma_i$
- A goal profile $\boldsymbol{\Gamma} = (\gamma_1, \dots, \gamma_n)$ collects agents' goals
- $\mathcal{L}^\star$ for $\star \in \{\wedge, \vee, \oplus\}$ defined as $\varphi := p \mid \neg p \mid \varphi \star \varphi$ are language restrictions on the goals

Colleagues ▲, ●, and ■ organize their next meeting.
They have to decide whether to meet in the morning (☼) or in the afternoon, to continue writing their paper (✍) or to talk about practicalities, and whether they'll meet at a local coffee shop (☕) or in their office.

The following are their propositional goals:

$\gamma_{▲}$ : ☼ ∧ ✍ ∧ ☕

$\gamma_{●}$ : ☼ ∧ ¬✍ ∧ ¬☕

$\gamma_{■}$ : (☼ ∧ ¬✍ ∧ ☕) ∨ (¬☼ ∧ ¬☕)

- $\text{Mod}(\gamma_{■}) = \{(101), (010), (000)\}$
- $m_{■}$(✍) $= (2, 1)$
- $\boldsymbol{\Gamma} = (\gamma_{▲}, \gamma_{●}, \gamma_{■})$

## Majoritarian Voting Rules

A goal-based voting rule is a collection of functions $F : (\mathcal{L}_\mathcal{I})^n \to \mathcal{P}(\{0,1\}^m) \setminus \emptyset$ for all $n$ and $m$ and $\mathcal{L}_\mathcal{I}$ a propositional language over $\mathcal{I}$.

$$EMaj(\boldsymbol{\Gamma})_j = 1 \ \textit{ iff } \ \sum_{i \in \mathcal{N}} \frac{m^1_{ij}}{|\text{Mod}(\gamma_i)|} \geq \left\lceil \frac{n+1}{2} \right\rceil$$

$TrueMaj(\boldsymbol{\Gamma}) = \Pi_{j \in \mathcal{I}} M(\boldsymbol{\Gamma})_j$ where, for $j \in \mathcal{I}$:

$$M(\boldsymbol{\Gamma})_j = \begin{cases} \{x\} & \text{if } \sum_{i \in \mathcal{N}} \frac{m^x_{ij}}{|\text{Mod}(\gamma_i)|} > \sum_{i \in \mathcal{N}} \frac{m^{1-x}_{ij}}{|\text{Mod}(\gamma_i)|} \\ \{0,1\} & \text{otherwise} \end{cases}$$

$$2sMaj(\boldsymbol{\Gamma}) = Maj(Maj(\gamma_1), \dots, Maj(\gamma_n))$$

## Manipulation

- Agent $i$ prefers the outcome of $F$ on $\boldsymbol{\Gamma}$ than on $\boldsymbol{\Gamma}'$ if and only if their satisfaction is higher on $\boldsymbol{\Gamma}$:
  $F(\boldsymbol{\Gamma}) \prec_i F(\boldsymbol{\Gamma}') \Leftrightarrow sat(i, F(\boldsymbol{\Gamma})) \geq sat(i, F(\boldsymbol{\Gamma}'))$

A rule $F$ is strategy-proof if and only if for all $\boldsymbol{\Gamma}$ there is no agent $i$ for whom $F(\boldsymbol{\Gamma}_{-i}, \gamma'_i) \prec_i F(\boldsymbol{\Gamma})$ for some $\gamma'_i$.

*Manipulation types*

unrestricted: $i$ can send any $\gamma'_i$ instead of $\gamma_i$

erosion: $i$ can only send $\gamma'_i$ s.t. $\text{Mod}(\gamma'_i) \subseteq \text{Mod}(\gamma_i)$

dilatation: $i$ can only send $\gamma'_i$ s.t. $\text{Mod}(\gamma_i) \subseteq \text{Mod}(\gamma'_i)$

**Summary of results**

| $\gamma_i \in$ | $\mathcal{L}$ E | $\mathcal{L}$ D | $\mathcal{L}^\wedge$ E | $\mathcal{L}^\wedge$ D | $\mathcal{L}^\vee$ E | $\mathcal{L}^\vee$ D | $\mathcal{L}^\oplus$ E | $\mathcal{L}^\oplus$ D |
|---|---|---|---|---|---|---|---|---|
| *EMaj* | M | M | SP | SP | M | SP | M | M |
| *TrueMaj* | M | M | SP | SP | M | SP | M | M |
| *2sMaj* | M | M | SP | SP | SP | SP | M | M |

*Erosion, Dilatation, Strategy-Proof, Manipulable*

| $\mathcal{N}$ | $\boldsymbol{\Gamma}$ | |
|---|---|---|
| ▲ | ☼ ∧ ✍ ∧ ☕ | (111) |
| ● | ☼ ∧ ¬✍ ∧ ¬☕ | (100) |
| ■ | (☼ ∧ ¬✍ ∧ ☕) ∨(¬☼ ∧ ¬☕) | (101) (010) (000) |
| *TrueMaj* | | (100) |

Can you find a manipulation for ■?

## Computational Complexity

How difficult it is to know if an agent can manipulate?

MANIP($F$) profile $\boldsymbol{\Gamma}$, agent $i$
• $\exists\, \gamma'_i$ such that $F(\boldsymbol{\Gamma}_{-i}, \gamma'_i) \prec_i F(\boldsymbol{\Gamma})$?

PP: problems solvable by a probabilistic TM in poly time, where TM says *yes* ⇔ a majority of computations accepts

MANIP(EMaj) and MANIP(2sMaj) are PP-hard.

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