Agents express **propositional goals** over **binary issues** to reach a collective decision. We study the **strategy-proofness** of three generalizations of the **majority rule**. We also study the **computational complexity** of finding a successful manipulation.

Framework

- A set N of n agents has to decide over a set I of m binary issues (no integrity constraint)
- Every agent *i* has a propositional formula *γ_i* as her goal, whose models are in the set Mod(*γ_i*)
- *m_i*(*j*) = (*m⁰_{ij}*, *m¹_{ij}*) indicates the number of 0s and 1s for issue *j* in the models of *γ_i*
- A goal profile $\Gamma = (\gamma_1, \ldots, \gamma_n)$ collects agents' goals
- \mathcal{L}^{\star} for $\star \in \{\land, \lor, \oplus\}$ defined as $\varphi := p | \neg p | \varphi \star \varphi$ are language restrictions on the goals

Colleagues \blacktriangle , •, and \blacksquare organize their next meeting. They have to decide whether to meet in the morning (\clubsuit) or in the afternoon, to continue writing their paper (\measuredangle) or to talk about practicalities, and whether they'll meet at a local coffee shop (\clubsuit) or in their office.

The following are their propositional goals:

$$\begin{array}{l} \gamma_{\blacktriangle} : \mathring{\nabla} \land \measuredangle \land \checkmark & & \\ \gamma_{\bullet} : \mathring{\nabla} \land \neg \measuredangle \land \neg \swarrow & \\ \gamma_{\blacksquare} : (\mathring{\nabla} \land \neg \measuredangle \land \checkmark) \lor (\neg \mathring{\nabla} \land \neg \And) \end{array}$$

• $Mod(\gamma_{\blacksquare}) = \{(101), (010), (000)\}$

$$\blacktriangleright \quad m_{\blacksquare}(\measuredangle) = (2,1)$$

 $\blacktriangleright \ \Gamma = (\gamma_{\blacktriangle}, \gamma_{\bullet}, \gamma_{\blacksquare})$

Majoritarian Voting Rules

A goal-based voting rule is a collection of functions

Manipulation

Agent *i* prefers the outcome of *F* on Γ than on Γ' if and only if their satisfaction is higher on Γ: $F(Γ) <_i F(Γ') ⇔ sat(i, F(Γ)) ≥ sat(i, F(Γ'))$

A rule *F* is strategy-proof if and only if for all Γ there is no agent *i* for whom $F(\Gamma_{-i}, \gamma'_i) \prec_i F(\Gamma)$ for some γ'_i .

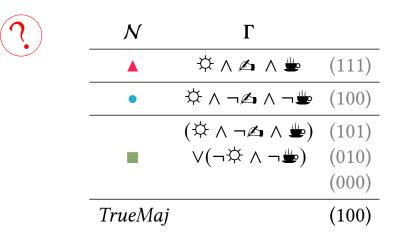
Manipulation types

unrestricted: *i* can send any γ'_i instead of γ_i erosion: *i* can only send γ'_i s.t. $Mod(\gamma'_i) \subseteq Mod(\gamma_i)$ dilatation: *i* can only send γ'_i s.t. $Mod(\gamma_i) \subseteq Mod(\gamma'_i)$

Summary of results

$\gamma_i \in$	L		<i>L</i> ^ E D		\mathcal{L}^{\vee}		\mathcal{L}^\oplus	
	Е	D	E	D	E	D	E	D
EMaj TrueMaj 2sMaj	Μ	Μ	SP	SP	Μ	SP	M	M
TrueMaj	Μ	Μ	SP	SP	Μ	SP	Μ	Μ
2sMaj	Μ	Μ	SP	SP	SP	SP	Μ	Μ

Erosion, Dilatation, Strategy-Proof, Manipulable



Can you find a manipulation for \blacksquare ?

Computational Complexity

 $F: (\mathcal{L}_{\mathcal{I}})^n \to \mathcal{P}(\{0,1\}^m) \setminus \emptyset$ for all *n* and *m* and $\mathcal{L}_{\mathcal{I}}$ a propositional language over \mathcal{I} .

$$\underline{EMaj}(\Gamma)_{j} = 1 \quad iff \quad \sum_{i \in \mathcal{N}} \frac{m_{ij}^{1}}{|\text{Mod}(\gamma_{i})|} \geq \lceil \frac{n+1}{2} \rceil$$
$$\underline{TrueMaj}(\Gamma) = \prod_{j \in \mathcal{I}} M(\Gamma)_{j} \text{ where, for } j \in \mathcal{I}:$$

$$M(\Gamma)_{j} = \begin{cases} \{x\} & \text{if } \sum_{i \in \mathcal{N}} \frac{m_{ij}^{x}}{|\text{Mod}(\gamma_{i})|} > \sum_{i \in \mathcal{N}} \frac{m_{ij}^{1-x}}{|\text{Mod}(\gamma_{i})|} \\ \{0, 1\} & \text{otherwise} \end{cases}$$

 $2sMaj(\Gamma) = Maj(Maj(\gamma_1), \ldots, Maj(\gamma_n))$

How difficult it is to know if an agent can manipulate?

MANIP(F) profile Γ , agent *i* • $\exists \gamma'_i$ such that $F(\Gamma_{-i}, \gamma'_i) \prec_i F(\Gamma)$?

PP: problems solvable by a probabilistic TM in poly time, where TM says yes ⇔ a majority of computations accepts

MANIP(EMaj) and MANIP(2sMaj) are PP-hard.

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