Judgment Aggregation in Dynamic Logic of Propositional Assignments

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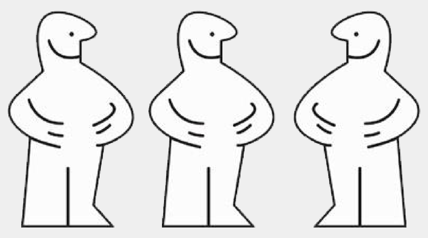
We translate problems of Binary Aggregation with Integrity Constraints (a model for Judgment Aggregation, where agents have binary opinions over issues and use aggregation procedures to reach a collective decision on them) into Dynamic Logic of Propositional Assignments (an instance of Propositional Dynamic Logic, where atomic programs change the truth value of variables). We focus on aggregation rules, properties of rules known as axioms, and the safety of the agenda problem.

Binary Aggregation with Integrity Constraints



A set of n *agents* express their *opinions* on a set of m *issues* to reach a *collective decision* on them.

$$\begin{array}{lllll} & & 1 \quad 2 \quad \dots \quad m & & \Leftarrow \textit{issues} \\ & B_1 & b^0_{11} \; b^1_{12} \; \dots \; b^0_{1m} & & \\ \text{A } \textit{profile } \boldsymbol{B} & B_2 & b^1_{21} \; b^1_{22} \; \dots \; b^0_{2m} & & \\ & B_i & \dots \; \dots \; b^1_{ij} \; \dots & & \Leftarrow \textit{individual ballot} \text{ of agent } i \\ & B_n & b^0_{n1} \; b^0_{n2} \; \dots \; b^1_{nm} & & \\ & F(\boldsymbol{B}) & \{(b^0_1 \; b^1_2 \; \dots \; b^1_m), \dots\} & & \Leftarrow \text{output of } \textit{aggregation rule} \end{array}$$

The *integrity constraint* IC is a propositional formula relating issues

The *models of* IC Mod(IC) is a set of all ballots making IC true

The *aggregation rule* is a function $F : \text{Mod(IC)}^n \to \mathcal{P}(\{0,1\}^m) \setminus \{\emptyset\}$

Dynamic Logic of Propositional Assignments

Atomic program $+p$ ($-p$) makes propositional variable p true (false).

Language

formulas $\varphi \ ::= \ p \mid \top \mid \bot \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\pi\rangle\varphi$

programs $\pi \ ::= \ +p \mid -p \mid \pi ; \pi \mid \pi \cup \pi \mid \pi^* \mid \varphi?$

for p in a countable set of propositional variables $\mathbb{P}$.

+ some *abbreviations* for programs ("if φ then π_1 else π_2", ...)

Interpretation

formulas set of *valuations* where the **formula** is true

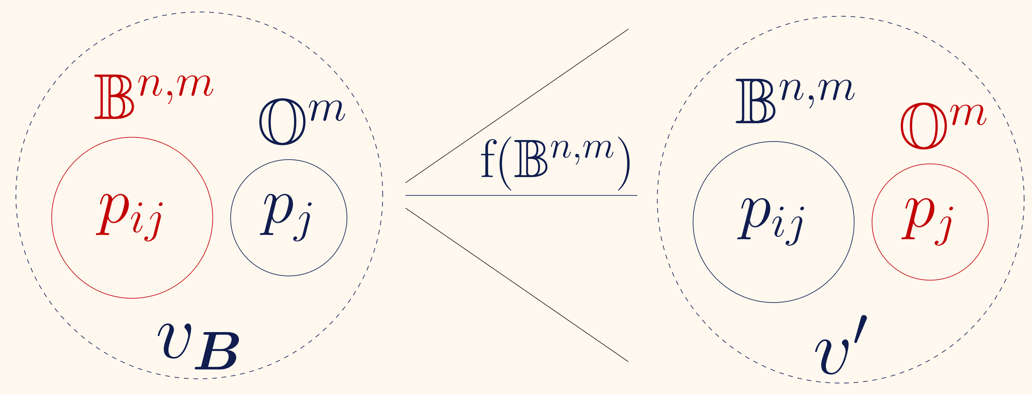
programs set of *pairs of valuations* such that you can go from the first to the second via the execution of the **program**

Translation of Judgment Aggregation into DL-PA

- sets of variables $\mathbb{B}^{n,m}$ and $\mathbb{O}^m$ for *input* and *output* of rules
- valuation $v_{\boldsymbol{B}}$ for the *values* of the input profile, such that

$$p_{ij} \in v_{\boldsymbol{B}} \iff b_{ij} = 1$$

  and valuation v' for the *values* of the output of F
- program $\mathrm{f}(\mathbb{B}^{n,m})$ to translate *rule* F
- formula IC written with variables in $\mathbb{O}^m$ for the *constraint*



A graphical representation of the translation into DL-PA.

Aggregation Rules

Aggregation rules ⇒ DL-PA *programs* (ensuring *correctness* of translation)

- **Simple rules:** dictatorship, majority, quota, ...

Dictatorship of agent i

BA | $\text{Dictatorship}_i(\boldsymbol{B}) \ = B_i$, for all $\boldsymbol{B}$

DL-PA | $\text{dictatorship}_i(\mathbb{B}^{n,m}) \ = \ ;_{1\le j\le m}(p_j \leftarrow p_{ij})$

- **Max/Min rules:** max. subagenda, min. number of atomic changes
- **Preference aggregation rules:** Kemeny, Slater, ...

Kemeny

BA | $\text{Kemeny}_{\text{IC}}(\boldsymbol{B}) = \ \text{argmin}_{B \models \text{IC}} \Sigma_{1\le i\le n} H(B, B_i)$

DL-PA | $\text{kemeny}_{\text{IC}}(\mathbb{B}^{n,m}) = \bigcup_{0\le d\le m} \big(\langle \text{flip}^1(\mathbb{O}^m)^d\rangle$
$(\text{MinD}(\mathbb{O}^m, \mathbb{B}^{n,m}, \text{IC}) \wedge \text{IC})? ; \text{flip}^1(\mathbb{O}^m)^d\big) ;$
$\text{MinD}(\mathbb{O}^m, \mathbb{B}^{n,m}, \text{IC}) \wedge \text{IC}?$

- **Representative voter rules:** average voter, majority voter, ...

Agenda Safety

IC properties ⇒ DL-PA *formulas* (ensuring *correctness* of translation)

- **median property**
- **simplified median property**
- **k-median property**
- **syntactic simplified median property**

IC properties linked to classes of rules whose outcomes will always satisfy IC.

Axioms

Axioms ⇒ DL-PA *formulas* (ensuring *correctness* of translation)

Single-profile: *the outcome* of F linked to structure of *a single* profile.

- unanimity, issue-neutrality, domain-neutrality, N-monotonicity

⇒ propositional logic

Unanimity

BA | U = For any $\boldsymbol{B}$, for all issues j and for $x \in \{0,1\}$, if $b_{ij} = x$ for all agents i then $F(\boldsymbol{B})_j = x$

DL-PA | U = $\bigwedge_{1\le j\le m} \Big(\big((\bigwedge_{1\le i\le n} p_{ij}) \to p_j \big) \wedge \big((\bigwedge_{1\le i\le n} \neg p_{ij}) \to \neg p_j \big) \Big)$

Multi-profile: *two outcomes* of F linked to structures of *multiple* profiles.

- independence, I-monotonicity, anonimity

⇒ DL-PA

Future Directions

▷ What about other rules, axioms, IC properties?

▷ What about the existing translation of DL-PA into propositional logic?

≫ It could be used for automated reasoning with SAT-solvers.

▷ What about other areas of Judgment Aggregation?

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