Goal-Based Collective Decisions
Axiomatics and Computational Complexity

Our agents express **propositional goals** over **binary issues** to reach a collective decision.
We adapt **axioms** and **rules** from Social Choice Theory, characterizing a generalization of the majority rule.
We study the **computational complexity** of finding the outcome of our rules (i.e., *winner determination*).

Agents ▲, ●, and ■ want to visit a city together. There are three points of interest: an ancient belfry (🔔), a music museum (♫), and the beach (≈).
▲ wants to visit everything, ● wants to go only to the museum, ■ wants to visit *a single place. . .*

Agents express goals with **propositional formulas**

$$\gamma_{▲} : 🔔 \wedge ♫ \wedge \approx$$
$$\gamma_{●} : \neg 🔔 \wedge ♫ \wedge \neg \approx$$
$$\gamma_{■} : (🔔 \wedge \neg ♫ \wedge \neg \approx) \vee (\neg 🔔 \wedge ♫ \wedge \neg \approx) \vee (\neg 🔔 \wedge \neg ♫ \wedge \approx)$$

Agents	Goal profile	
▲	$(🔔 \wedge ♫ \wedge \approx)$	(111)
●	$(\neg 🔔 \wedge \neg ♫ \wedge \approx)$	(001)
■	$(🔔 \wedge \neg ♫ \wedge \neg \approx) \vee$	(100)
	$(\neg 🔔 \wedge ♫ \wedge \neg \approx) \vee$	(010)
	$(\neg 🔔 \wedge \neg ♫ \wedge \approx)$	(001)

What is the output of the different rules?

Framework and Notation

- A set $\mathcal{N}$ of n agents has to decide over a set $\mathcal{I}$ of m binary issues (there are **no integrity constraints**)
  - $\mathcal{N} = \{▲, ●, ■\}$ and $\mathcal{I} = \{🔔, ♫, \approx\}$
- Every agent i has a propositional formula γ_i as her goal, whose models are in the set $\mathrm{Mod}(\gamma_i)$
- Vector $m_i(j) = (m^0_{ij}, m^1_{ij})$ indicates the number of 0s and 1s for issue j in the models of γ_i
  - $\mathrm{Mod}(\gamma_{■}) = \{(100), (010), (001)\}$
  - $m_{■}(♫) = (2, 1)$
- A goal profile $\mathbf{\Gamma} = (\gamma_1, \dots, \gamma_n)$ collects agents' goals
- A goal-based voting rule is a collection of functions $F : (\mathcal{L}_{\mathcal{I}})^n \longrightarrow \mathcal{P}(\{0,1\}^m) \setminus \emptyset$ for all n and m, where $\mathcal{L}_{\mathcal{I}}$ is a propositional language over $\mathcal{I}$.
- F is resolute if it always returns a singleton (irresolute otherwise)

Goal-based Voting Rules

$$Conj_v(\mathbf{\Gamma}) = \begin{cases} \mathrm{Mod}(\gamma_1 \wedge \cdots \wedge \gamma_n) & \text{if non-empty} \\ \{v\} \text{ for } v \in \{0,1\}^m & \text{otherwise} \end{cases}$$

$$Approval(\mathbf{\Gamma}) = \underset{v \in \mathrm{Mod}(\bigvee_{i \in N} \gamma_i)}{\arg\max} |\{i \in N \mid v \in \mathrm{Mod}(\gamma_i)\}|$$

1. $EMaj(\mathbf{\Gamma})_j = 1$ *iff* $\sum_{i \in N} \frac{m^1_{ij}}{|\mathrm{Mod}(\gamma_i)|} \geq \frac{n}{2}$
2. $TrueMaj(\mathbf{\Gamma}) = \Pi_{j \in \mathcal{I}} M(\mathbf{\Gamma})_j$ where, for $j \in \mathcal{I}$:

$$M(\mathbf{\Gamma})_j = \begin{cases} \{x\} & \text{if } \sum_{i \in N} \frac{m^x_{ij}}{|\mathrm{Mod}(\gamma_i)|} > \sum_{i \in N} \frac{m^{1-x}_{ij}}{|\mathrm{Mod}(\gamma_i)|} \\ \{0,1\} & \text{otherwise} \end{cases}$$

3. $2sMaj(\mathbf{\Gamma}) = Maj(Maj(\gamma_1), \dots, Maj(\gamma_n))$

Axiomatics - Characterization

Anonymity (A): *Agents' (goals)* are equally important
Neutrality (N): *Issues* are equally important
Independence (I): *Each issue j* is decided by a function f_j
Unanimity (U): Result follows *agents' unanimous choice*
Positive responsiveness (PR): *Adding (deleting) support for an issue* when the result is equally irresolute or favoring acceptance (rejection), gives a result strictly favoring acceptance (rejection)
Egalitarianism (E): Every model of a goal has a *weight proportional to the total number* of models of the goal
Duality (D): Rule isn't *biased for acceptance/rejection*

A goal-based voting rule satisfies (E), (I), (A), (N), (PR), (U) and (D) if and only if it is *TrueMaj*.

Similar to results for majority in Judgment Aggregation.

Computational Complexity

We study the complexity to compute the result of rules.

WinDet(F) profile $\mathbf{\Gamma}$, issue j
• $F(\mathbf{\Gamma})_j = 1$?

WinDet*(F) profile $\mathbf{\Gamma}$, set $S \subseteq \mathcal{I}$, $\rho : S \longrightarrow \{0,1\}$
• $\exists\, v \in F(\mathbf{\Gamma})$ with $v(j) = \rho(j)$ for $j \in S$?

Θ^p_2: problems solvable in poly time with O(log n) queries to an NP oracle

PP: problems solvable by a probabilistic TM in poly time, with error probability < 1/2

WinDet*(Conj) is NP-hard.
WinDet*(Approval) is Θ^p_2-complete.
WinDet of majorities is PP-hard.

Reductions from: SAT, Max-Model and Maj-SAT-p.

Arianna Novaro[1], Umberto Grandi[1], Dominique Longin[2], Emiliano Lorini[2]
{arianna.novaro, umberto.grandi, dominique.longin, emiliano.lorini}@irit.fr
[1]IRIT, University of Toulouse. [2] IRIT, CNRS, Toulouse.