Goal-based Collective Decisions

Axiomatics and Computational Complexity

IJCAI-2018

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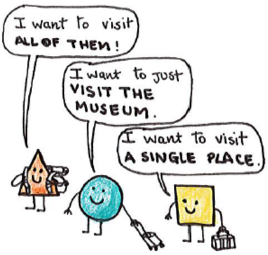
A group of agents wants to visit Stockholm



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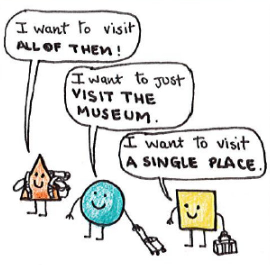

$\Rightarrow$ use propositional formulas

Framework

- n agents decide on m binary issues
  - $\{\blacktriangle, \bullet, \blacksquare\}$ and $\{c, m, a\}$
- agent i has for goal a propositional formula γ_i, whose models are in the set Mod(γ_i)
  - $\gamma_{\blacksquare} = (c \wedge \neg m \wedge \neg a) \vee (\neg c \wedge m \wedge \neg a) \vee (\neg c \wedge \neg m \wedge a)$
  - Mod($\gamma_{\blacksquare}$) $= \{(100), (010), (001)\}$
- a goal-profile $\mathbf{\Gamma} = (\gamma_1, \ldots, \gamma_n)$ contains all agents' goals
  - $(\gamma_{\blacktriangle}, \gamma_{\bullet}, \gamma_{\blacksquare})$

Goal-based Voting Rules

A goal-based voting rule is a collection of functions for all n and m:

$$F : (\mathcal{L}_{\mathcal{I}})^n \to \mathcal{P}(\{0,1\}^m) \setminus \emptyset$$

- $(\gamma_{▲}, \gamma_{●}, \gamma_{■}) \mapsto \{(101), (000)\}$

A resolute rule returns singleton on all profiles (else, irresolute)

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Conjunction: If the *conjunction of the agents' goals* is satisfiable, pick the common models (else, a default)

Approval: All models *satisfying the maximal number of goals*

Issue-wise Majorities

1. $EMaj(\mathbf{\Gamma})_j = 1$ *iff* $\left(\sum_{i \in \mathcal{N}} \frac{m^1_{ij}}{|\mathsf{Mod}(\gamma_i)|}\right) \geq \frac{n}{2}$
2. $TrueMaj(\mathbf{\Gamma}) = \Pi_{j \in \mathcal{I}} M(\mathbf{\Gamma})_j$ with, for $j \in \mathcal{I}$:

$$M(\mathbf{\Gamma})_j = \begin{cases} \{x\} & \text{if } \sum_{i \in \mathcal{N}} \frac{m^x_{ij}}{|\mathsf{Mod}(\gamma_i)|} > \sum_{i \in \mathcal{N}} \frac{m^{1-x}_{ij}}{|\mathsf{Mod}(\gamma_i)|} \\ \{0,1\} & \text{otherwise} \end{cases}$$

3. $2sMaj(\mathbf{\Gamma}) = Maj(Maj(\gamma_1), \ldots, Maj(\gamma_n))$

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3. $2sMaj(\mathbf{\Gamma}) = Maj(Maj(\gamma_1), \ldots, Maj(\gamma_n))$

▲	$(c \wedge m \wedge a)$	(111)
●	$(\neg c \wedge \neg m \wedge a)$	(001)
■	$(c \wedge \neg m \wedge \neg a) \vee (\neg c \wedge m \wedge \neg a) \vee (\neg c \wedge \neg m \wedge a)$	(100) (010) (001)

Axiomatics

We axiomatically characterize the $TrueMaj$ rule, adapting axioms from Social Choice Theory.

A rule F satisfies Anonymity, Neutrality, Independence, Unanimity, Positive Responsiveness, Egalitarianism and Duality

if and only if

F is $TrueMaj$.

Computational Complexity

We study the *winner determination* problem for our rules (how hard is it to compute the result).

- $\Theta_2^p = \mathrm{P}^{\mathrm{NP}[\log n]}$
- PP = Probabilistic Polynomial time

$\textsc{WinDet}^\star$ of approval is Θ_2^p-complete.

$\textsc{WinDet}$ of majorities is PP-hard.

Conclusions

- Introduction of the framework of goal-based voting
- Definition of multiple voting rules, with
  - an axiomatic characterisation of the $TrueMaj$ rule
  - a study of their computational complexity
- Tension between resoluteness and biasedness

≫ Open questions in complexity

≫ Agents behaving strategically