Relaxing Exclusive Control in Boolean Games

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Scenario 1: Friends Organize a Potluck

meat

"If we have steak I want red wine."

wine

"I hope we eat steak or herring."

fish

"I hate herring and I like white wine."

Scenario 2: Friends Organize a Visit

Decide together which places to visit.

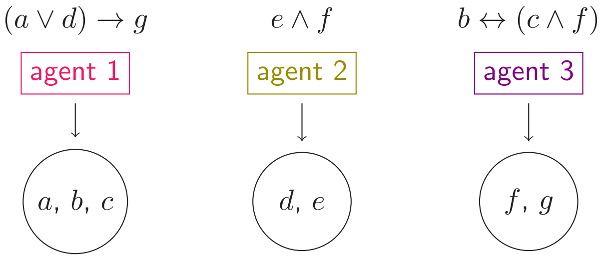
Should we go check out the bridge?
Should we go see the clock?
Should we visit the castle?

Talk Outline

1. Games of Propositional Control
   Boolean Games and Iterated Boolean Games
2. Strategics Abilities in Logic
   Concurrent Game Structures with Exclusive Control
   Concurrent Game Structures with Shared Control
3. Main Results
   Relationship between Exclusive and Shared Control
   Computational Complexity
4. Conclusions

Games of Propositional Control

Boolean Games, Intuitively



Harrenstein, van der Hoek, Meyer and Witteveen. *Boolean games*. TARK-2001.
Bonzon, Lagasquie-Schiex, Lang and Zanuttini. *Boolean games revisited*. ECAI-2006.

Boolean Games, Formally

A Boolean Game is a tuple $G = (N, \Phi, \pi, \mathbf{\Gamma})$ such that:

- $N = \{1, \ldots, n\}$ is a set of agents
- Φ is a finite set of variables
- $\pi : N \to 2^{\Phi}$ is a control function (a partition of Φ)
- $\mathbf{\Gamma} = \{\gamma_1, \ldots, \gamma_n\}$ is a set of **propositional** formulas over Φ

$$N = \{1, 2, 3\}$$
$$\Phi = \{a, b, c, d, e, f, g\}$$
$$\pi(1) = \{a, b, c\},\ \pi(2) = \{d, e\},\ \pi(3) = \{f, g\}$$
$$\mathbf{\Gamma} = \{\ (a \vee d) \to g\ ,\ e \wedge f\ ,\ b \leftrightarrow (c \wedge f)\ \}$$

Strategies and Utilities for Boolean Games

A strategy σ_i is an assignment to the variables in $\pi(i)$.
A strategy profile is a tuple $\boldsymbol{\sigma} = (\sigma_1, \dots, \sigma_n)$: a valuation on Φ.
The (binary) utility of agent i is 1 if $\boldsymbol{\sigma} \models \gamma_i$, and 0 otherwise.

$\pi(1) = \{a, b, c\}$, $\pi(2) = \{d, e\}$, $\pi(3) = \{f, g\}$

$\sigma_1(a) = \sigma_1(b) = 1$, $\sigma_1(c) = 0$ $\quad\quad$ $\sigma_1 = \{a, b\}$

$\sigma_2(d) = 0$, $\sigma_2(e) = 1$ $\quad\quad$ $\sigma_2 = \{e\}$

$\sigma_3(f) = \sigma_3(g) = 1$ $\quad\quad$ $\sigma_3 = \{f, g\}$

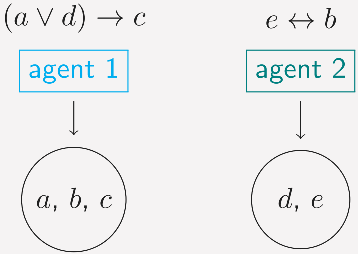
Which are the utilities of the agents?

$$\boldsymbol{\Gamma} = \{ (a \vee d) \rightarrow g, e \wedge f, b \leftrightarrow (c \wedge f) \}$$

Winning Strategies

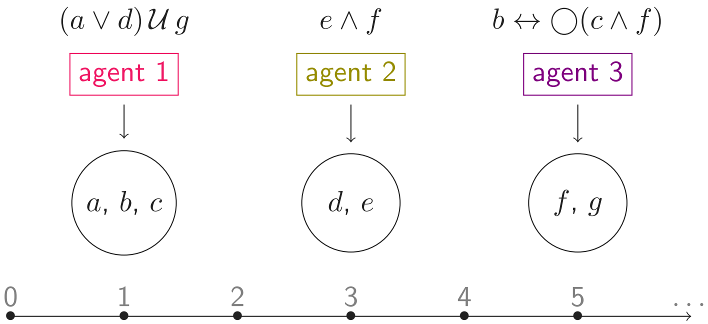
$\sigma_{-i} = (\sigma_1, \ldots, \sigma_{i-1}, \sigma_{i+1}, \ldots, \sigma_n)$ is the projection of $\boldsymbol{\sigma}$ on $N \setminus \{i\}$

A winning strategy σ_i for i is such that $(\sigma_{-i}, \sigma_i) \models \gamma_i$ for all σ_{-i}.



A winning strategy for agent 1? And for agent 2?

Iterated Boolean Games, Intuitively



Gutierrez, Harrenstein, Wooldridge. *Iterated Boolean Games*. Information and Computation 242:53-79. (2015).

Iterated Boolean Games, Formally

An iterated Boolean Game is a tuple $G = (N, \Phi, \pi, \mathbf{\Gamma})$ such that:

- $N = \{1, \dots, n\}$ is a set of agents
- Φ is a finite set of variables
- $\pi : N \to 2^{\Phi}$ is a control function (a partition of Φ)
- $\mathbf{\Gamma} = \{\gamma_1, \dots, \gamma_n\}$ is a set of **LTL** formulas over Φ

We assume that agents have memory-less strategies
$=$ their choice of action depends on the *current* state only.

Strategic Abilities in Logic

What Can Agents Do?
ATL* Syntax

Alternating-time Temporal Logic (*) allows us to talk about the strategic abilities of the agents, when time is involved.

$$\begin{array}{lcl} \varphi & ::= & p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\langle C \rangle\rangle \psi \\ \psi & ::= & \varphi \mid \neg\psi \mid \psi \vee \psi \mid \bigcirc\psi \mid \psi\,\mathcal{U}\,\psi \end{array}$$

$\langle\langle C \rangle\rangle \psi$ agents in C can enforce ψ, regardless of actions of others

$\bigcirc\psi$ ψ holds at the next step

$\psi_1\,\mathcal{U}\,\psi_2$ ψ_2 holds in the future, and until then ψ_1 holds

Interpreted over Concurrent Game Structures (CGS), such as ...

Concurrent Game Structures with Exclusive Propositional Control

A CGS-EPC is a tuple $\mathcal{G} = (N, \Phi_1, \ldots, \Phi_n, S, d, \tau)$ where:

- $N = \{1, \ldots, n\}$ is a set of agents
- $\Phi = \Phi_1 \cup \cdots \cup \Phi_n$ is a set of variables (partition)
- $S = 2^\Phi$ is the set of states, i.e., all valuations over Φ
- $d : N \times S \to (2^\mathcal{A} \setminus \emptyset)$, for $\mathcal{A} = 2^\Phi$, is the protocol function, such that $d(i, s) \subseteq \mathcal{A}_i$ for $\mathcal{A}_i = 2^{\Phi_i}$
- $\tau : S \times \mathcal{A}^n \to S$ is the transition function, such that $\tau(s, \alpha_1, \ldots, \alpha_n) = \bigcup_{i \in N} \alpha_i$

Belardinelli, Herzig. *On Logics of Strategic Ability based on Propositional Control*. IJCAI-2016.

Example of CGS-EPC: Friends Organize a Potluck

- $N = \{1, 2, 3\}$
- $\Phi = \Phi_1 \cup \Phi_2 \cup \Phi_3 = \{\text{wine}\} \cup \{\text{steak}\} \cup \{\text{herring}\}$
- $S = \{\emptyset, \{\text{wine}\}, \{\text{wine}, \text{steak}\}, \{\text{wine}, \text{steak}, \text{herring}\}, \dots\}$
- for any $s \in S$, $d(1, s) = \{\emptyset, \{\text{wine}\}\}$, $d(2, s) = \{\emptyset, \{\text{steak}\}\}$, $d(3, s) = \{\emptyset, \{\text{herring}\}\}$
- $\tau(s, \alpha_1, \alpha_2, \alpha_3) = \alpha_1 \cup \alpha_2 \cup \alpha_3$
  - $\tau(s, \{\text{wine}\}, \{\text{steak}\}, \emptyset) = \{\text{wine}, \text{steak}\} = s'$

Concurrent Game Structures with Shared Propositional Control

A CGS-SPC is a tuple $\mathcal{G} = (N, \Phi_0, \ldots, \Phi_n, S, d, \tau)$ where:

- N, S, and d are defined as for CGS-EPC
- $\Phi = \Phi_0 \cup \Phi_1 \cup \cdots \cup \Phi_n$ is a set of variables
- $\tau : S \times \mathcal{A}^n \to S$ is the transition function

Belardinelli, Grandi, Herzig, Longin, Lorini, Novaro, Perrussel. *Relaxing Exclusive Control in Boolean Games*. TARK-2017.

Example of CGS-SPC: Friends Organize a Visit

- $N = \{1, 2, 3\}$
- $\Phi = \Phi_1 = \Phi_2 = \Phi_3 = \{\text{bridge}, \text{clock}, \text{castle}\}$
- $S = \{\emptyset, \{\text{bridge}\}, \{\text{bridge}, \text{clock}\}, \{\text{clock}, \text{castle}\}, \dots\}$
- for any $s \in S$, $d(1, s) = d(2, s) = d(3, s) = S$
- $p \in \tau(s, \alpha_1, \alpha_2, \alpha_3)$ if and only if $|\{i \in N \mid p \in \alpha_i\}| \geq 2$
  - $\tau(s, \{\text{bridge, castle}\}, \{\text{clock}\}, \{\text{castle}\}) = \{\text{castle}\} = s'$

What Can Agents Do? ATL* Semantics

- $\lambda = s_0 s_1 \dots$ is a path if, for all $k \geq 0$, $\tau(s_k, \alpha) = s_{k+1}$ such that $\alpha = (\alpha_1, \dots, \alpha_n)$ and $\alpha_i \in d(i, s_k)$ for $i \in N$
- $\mathit{out}(s, \boldsymbol{\sigma}_C) = \{\lambda \mid s_0 = s$ and, for $k \geq 0$, there is α such that $\boldsymbol{\sigma}_C(i)(s_k) = \alpha_i$ for all $i \in C$ and $\tau(s_k, \alpha) = s_{k+1}\}$

$$
\begin{array}{rll}
(\mathcal{G}, s) \models p & \text{iff} & p \in s \\
(\mathcal{G}, s) \models \langle\!\langle C \rangle\!\rangle \psi & \text{iff} & \text{for some } \boldsymbol{\sigma}_C, \text{ for all } \lambda \in \mathit{out}(s, \boldsymbol{\sigma}_C), (\mathcal{G}, \lambda) \models \psi \\
(\mathcal{G}, \lambda) \models \varphi & \text{iff} & (\mathcal{G}, \lambda[0]) \models \varphi \\
(\mathcal{G}, \lambda) \models \bigcirc \varphi & \text{iff} & (\mathcal{G}, \lambda[1, \infty]) \models \varphi \\
(\mathcal{G}, \lambda) \models \varphi \, \mathcal{U} \, \psi & \text{iff} & \text{there is } t' \geq 0 \text{ such that } \big((\mathcal{G}, \lambda[t', \infty]) \models \psi \text{ and} \\
& & \text{for all } 0 \leq t'' < t' : (\mathcal{G}, \lambda[t'', \infty]) \models \varphi\big)
\end{array}
$$

Iterated Boolean Games as CGS

An Iterated Boolean Game is a tuple $(\mathcal{G}, \gamma_1, \ldots, \gamma_n)$ such that

- $\mathcal{G}$ is a CGS-EPC where $d(i,s) = \mathcal{A}_i$ for every $i \in N$ and $s \in S$
- for every $i \in N$ the goal γ_i is an LTL formula

An Iterated Boolean Game with shared control is a tuple $(\mathcal{G}, \gamma_1, \ldots, \gamma_n)$ such that

- $\mathcal{G}$ is a CGS-SPC
- for every $i \in N$ the goal γ_i is an LTL formula

We can also express influence games and aggregation games.

Grandi, Lorini, Novaro, Perrussel. *Strategic Disclosure of Opinions on a Social Network*. AAMAS-2017.
Grandi, Grossi, Turrini. *Equilibrium Refinement through Negotiation in Binary Voting*. IJCAI-2015.

Main Results

Exclusive and Shared Control Structures

A CGS-SPC can be simulated by a CGS-EPC.

- ● ○ ○ Define a corresponding CGS-EPC from a given CGS-SPC
- ● ● ○ Define a translation function tr within ATL*
- ● ● ● Show that the CGS-SPC satisfies φ if and only if the corresponding CGS-EPC satisfies $tr(\varphi)$

● ○ ○ | The corresponding CGS-EPC

Shared control (CGS-SPC) $\mathcal{G} = (N, \Phi_0, \ldots, \Phi_n, S, d, \tau)$

Exclusive control (CGS-EPC) $\mathcal{G}' = (N', \Phi'_1, \ldots, \Phi'_n, S', d', \tau')$

N' = adding a dummy agent

Φ' = adding a $turn$ variable and local copies of variables in Φ

- agent i controls her copies; dummy controls Φ and $turn$

S' = all valuations over Φ'

d' = depends on the truth value of $turn$ variable:
agents act when $turn$ false; dummy acts when $turn$ true

τ' = updates according to agents' actions

Example and Graphical Representation

$$
\begin{aligned}
N = \{1, 2\} \quad &\mapsto \quad N' = \{1, 2, *\} \\
\Phi_1 = \{p\},\ \Phi_2 = \{p, q\} \quad &\mapsto \quad \Phi_* = \{p, q, turn\}, \\
& \quad \Phi_1 = \{c_{1p}\},\ \Phi_2 = \{c_{2p}, c_{2q}\}
\end{aligned}
$$

$$
\begin{tikzcd}
\lambda[0] \arrow[rr, "(\alpha_1 \dots \alpha_n)[0]"] \arrow[d, dashed] & & \lambda[1] \arrow[rr, "(\beta_1 \dots \beta_n)[1]"] \arrow[d, dashed] & & \lambda[2] \arrow[r, "(\delta_1 \dots \delta_n)[2]"] \arrow[d, dashed] & \cdots \\
\lambda'[0] \arrow[r, "\alpha'_1 \dots \alpha'_n", "+turn"'] & \lambda'[1] \arrow[r, "\emptyset", "\tau(\lambda'[1]_{|\Phi}, \alpha)"'] & \lambda'[2] \arrow[r, "\beta'_1 \dots \beta'_n", "+turn"'] & \lambda'[3] \arrow[r, "\emptyset", "\tau(\lambda'[3]_{|\Phi}, \beta)"'] & \lambda'[4] \arrow[r, "\delta'_1 \dots \delta'_n", "+turn"'] & \cdots
\end{tikzcd}
$$

● ○ ○ | The corresponding CGS-EPC

Shared control (CGS-SPC) $\mathcal{G} = (N, \Phi_0, \dots, \Phi_n, S, d, \tau)$

Exclusive control (CGS-EPC) $\mathcal{G}' = (N', \Phi'_1, \dots, \Phi'_n, S', d', \tau')$

$N' = N \cup \{*\}$

$\Phi' = \Phi \cup \{turn\} \cup \{c_{ip} \mid i \in N$ and $p \in \Phi_i\}$

- $\Phi'_i = \{c_{ip} \in \Phi' \mid p \in \Phi_i\}$; $\Phi'_* = \{turn\} \cup \Phi$

$S' = 2^{\Phi'}$

$\neg turn$ $d'(i, s') = \{\alpha'_i \in \mathcal{A}'_i \mid \alpha_i \in d(i, s)\}$ $d'(*, s') = +turn$

$turn$ $d'(i, s') = \emptyset$ $d'(*, s') = \tau(s, \alpha)$ for $\alpha_i(p) = s'(c_{ip})$

$\tau' = \bigcup_{i \in N'} \alpha'_i$

● ● ○ | Translation tr within ATL*

For $p \in \Phi$, $C \subseteq N$ and χ, χ' either *state* or *path* formulas:

$$\begin{array}{lcl} tr(p) & = & p \\ tr(\neg\chi) & = & \neg tr(\chi) \\ tr(\chi \vee \chi') & = & tr(\chi) \vee tr(\chi') \\ tr(\bigcirc\chi) & = & \bigcirc \bigcirc tr(\chi) \\ tr(\chi \,\mathcal{U}\, \chi') & = & tr(\chi)\,\mathcal{U}\, tr(\chi') \\ tr(\langle\!\langle C \rangle\!\rangle\chi) & = & \langle\!\langle C \rangle\!\rangle tr(\chi) \end{array}$$

- $tr(p \vee q) = tr(p) \vee tr(q) = p \vee q$
- $tr(\bigcirc(p \vee q)) = \bigcirc \bigcirc tr(p \vee q) = \ldots = \bigcirc \bigcirc (p \vee q)$

Intermezzo: Hidden Machinery

- × The CGS-EPC has more variables than the original CGS-SPC
  - ✓ For state s in the CGS-SPC, define a canonical state in the CGS-EPC that agrees with s on Φ and everything else is false
- × There are many paths λ' in the CGS-EPC that could be associated to a path λ in the original CGS-SPC
  - ✓ Associate paths from the CGS-SPC and the CGS-EPC; then, define the canonical paths (starting from the canonical state)
- × Analogously, the strategies of CGS-SPC and CGS-EPC differ
  - ✓ For each joint strategy in the CGS-SPC there is an associated one in the CGS-EPC; and viceversa

Main Result

Given a CGS-SPC $\mathcal{G}$, the corresponding CGS-EPC $\mathcal{G}'$ is such that for all state-formulas φ and all path-formulas ψ in ATL*:

$$\begin{array}{llll} \text{for all } s \in S & (\mathcal{G}, s) \models \varphi & \text{if and only if} & (\mathcal{G}', s'_*) \models tr(\varphi) \\ \text{for all } \lambda \text{ of } \mathcal{G} & (\mathcal{G}, \lambda) \models \psi & \text{if and only if} & (\mathcal{G}', \lambda'_*) \models tr(\psi) \\ & & & \text{for any } \lambda'_* \end{array}$$

Proof. By induction on the structure of formulas φ and ψ.

Computational Complexity of CGS-SPC

Model-checking of ATL* in CGS-SPC is PSPACE-complete.

Proof. For membership use the PSPACE algorithm for ATL* on general CGS. For hardness, satisfiability of LTL formula φ can be reduced to model-checking $\langle\langle 1 \rangle\rangle \varphi$ on a CGS-SPC with one agent.

If G is an IBG with shared control, determining whether i has a winning strategy is in PSPACE.

Proof. We have to check that $\langle\langle i \rangle\rangle \gamma_i$ holds.

Conclusions

Conclusions

- We defined a new class of concurrent game structures (CGS) where agents may have shared control over variables
- We showed that they can be (polynomially) "simulated" within the class of CGS with exclusive control
- We showed that the complexity of the model-checking problem of ATL* on CGS-SPC is PSPACE-complete